Titolo del corso: Continuous constraint satisfaction and optimization problems

Docente: Pierfrancesco Urbani

Ore frontali di lezione: 20h

Periodo di lezione: Marzo — Giugno 2026

Settore/i disciplinare del corso: MAT07

Tipologia di corso: Avanzato

Modalità di verifica dell'apprendimento: Esame orale

Abstract del corso: Constraint satisfaction problems (CSP) are central in computational complexity theory. They are defined through a large number of degrees of freedom which are subjected to a large number of constraints. Finding solutions to all the constraints may be simple or hard depending on the control parameters of the problem. In recent years statistical physics techniques have been developed to study random CSP, namely CSP extracted from an ensemble of problems. In these lectures, I will focus on a particular class of such CSP that are defined out of Continuous (real) degrees of freedom (CCSP). These problems have become very popular nowadays due to the fact CCSPs appear in a wide variety of fields: artificial neural networks, glasses, and granular materials, the physics of biological tissues, packing problems in mathematics, and field theory in high energy physics. In these lectures, I will discuss simple paradigmatic models of non-convex CCSP in high dimension, introduce general methods to study the landscape of solutions and their concentration properties, and consider the algorithmic problem of finding solutions via general classes of algorithms (gradient descent and variants, message passing).

Programma del corso:

- Introduction to constraint satisfaction problems and computational complexity. Discrete CSP.
- Continuous constraint satisfaction problems: motivations, similarities and differences with discrete CSP.
- CCSP in high dimension: simple models.
- Statistical Physics approach to the space of solutions of random CCSP. Satisfiability transition and critical behavior.
- Planted CCSP: high dimensional inference and learning
- Dynamical mean field theory of gradient descent search of solutions.
- Sampling the solution space: efficient algorithms and hard phases.

Methods: replica method, message passing algorithms, dynamical mean field theory, large deviations.